**DAILY ASSESSMENT FORMAT**

|  |  |  |  |
| --- | --- | --- | --- |
| **Date:** | **17-07-2020** | **Name:** | **Bhavith** |
| **Course:** | **Coursera** | **USN:** | **4AL17EC009** |
| **Topic:** | **Mapping matrices and applying data** | **Semester & Section:** | **6th,A** |
| **Github Repository:** | **Bhavith-Online-Courses** |  |  |

|  |
| --- |
| **FORENOON SESSION DETAILS** |
| **Image of session**  **Screenshot (202)**  **Screenshot (203)** |
| **Report – Report can be typed or hand written for up to two pages.**  **In mathematics, a linear map (also called a linear mapping, linear transformation or, in some contexts, linear function) is a mapping V → W between two modules (for example, two vector spaces) that preserves (in the sense defined below) the operations of addition and scalar multiplication.** **Vectorization of Matrices** **The term  vectorization of a matrix  denotes a linear transformation which converts a matrix with mm rows  and nn columns into a column vector of size mn. mn.**  **Consider a matrix A=[aij]m×n∈Mm×n(K), A=[aij]m×n∈Mm×n(K),  where Mm×n(K) Mm×n(K)  is the vector space of m×n m×n  matrices over a field K. K.  Let Em×n={Eij:  i=1,…,m; j=1,…,n} Em×n={Eij:  i=1,…,m; j=1,…,n}  be the standard basis of Mm×n(K), Mm×n(K),  composed of matrices Eij Eij  with exactly one non-zero entry,  equal to unity,  in the ii-th row and jj-th column:**  **(Eij)|kl =  δik δjl,i,k=1,2,…,m,j,l=1,2,…,n.(Eij)|kl =  δik δjl,i,k=1,2,…,m,j,l=1,2,…,n.**  **The matrix A A  may be vectorized in two ways:**  **by juxtaposing the consecutive rows of the matrix next to each other  and taking the transpose of the obtained long “multi-row”:**  **Λmn(A) := [a11 a12 … a1n  a21 a22 … a2n  …  am1 am2 … amn]T.Λmn(A) := [a11 a12 … a1n  a21 a22 … a2n  …  am1 am2 … amn]**  **The vector Λmn(A) Λmn(A)  is the column of coordinates of matrix A A  in the ordered basis**  **Erowm×n = ( E11, E12, …, E1n,  E21, E22, …, E2n,  …,  Em1, Em2, …, Emn).Em×nrow = ( E11, E12, …, E1n,  E21, E22, …, E2n,  …,  Em1, Em2, …, Emn).**  **by stacking the columns of the matrix on top of one another:**  **Vmn(A) := [a11 a21 … am1  a12 a22 … am2  …  a1n a2n … amn]T.Vmn(A) := [a11 a21 … am1  a12 a22 … am2  …  a1n a2n … amn]T.**  **The vector Vmn(A) Vmn(A)  is the column of coordinates of matrix A A  in the ordered basis**  **Ecolm×n = ( E11, E21, …, Em1,  E12, E22, …, Em2,  …,  E1n, E2n, …, Emn).Em×ncol = ( E11, E21, …, Em1,  E12, E22, …, Em2,  …,  E1n, E2n, …, Emn).**  **In literature [[1]](http://visual.icse.us.edu.pl/LA/tensor_product_vectorization.html" \l "id2)    the vector Vmn(A) Vmn(A)  is also denoted by vec(A).vec(A).**  ****Example.**  The standard basis of the vector space  M2×3(R)  M2×3(R)  is composed of the matrices**  **E11 = [100000],E21 = [010000],E12 = [001000],E22 = [000100],E13 = [000010],E23 = [000001],E11 = [100000],E12 = [010000],E13 = [001000],E21 = [000100],E22 = [000010],E23 = [000001],**  **hence the ordered bases are:**  **Erow2×3 = ( E11, E12, E13,  E21, E22, E23 ),Ecol2×3 = ( E11, E21, E12,  E22, E13, E23 ).E2×3row = ( E11, E12, E13,  E21, E22, E23 ),E2×3col = ( E11, E21, E12,  E22, E13, E23 ).**  **For the matrix  A = [142536]   A = [123456]   we get   Λ23(A) = ⎡⎣⎢⎢⎢⎢⎢⎢⎢⎢123456⎤⎦⎥⎥⎥⎥⎥⎥⎥⎥,  V23(A) = ⎡⎣⎢⎢⎢⎢⎢⎢⎢⎢142536⎤⎦⎥⎥⎥⎥⎥⎥⎥⎥.  Λ23(A) = [123456],  V23(A) = [142536].**  **For  A∈Mm×n(K)  A∈Mm×n(K)  obviously  Vmn(A) = Λnm(AT), Vmn(A) = Λnm(AT),  Λmn(A) = Vnm(AT). Λmn(A) = Vnm(AT).**  **Using the Kronecker product, matrix multiplication can be expressed as a linear transformation of vectorized matrices. With this end in view, assume that  A=[aij]m×p,  A=[aij]m×p,   B=[bij]p×n  B=[bij]p×n  and  C≡AB=[cij]m×n  C≡AB=[cij]m×n  are matrices over a field  K.  K.  Then**  **(1)**  **cij = ∑k=1p aik bkj,i=1,2,…,m; j=1,2,…,n.cij = ∑k=1p aik bkj,i=1,2,…,m; j=1,2,…,n.**  **Equation [(1)](http://visual.icse.us.edu.pl/LA/tensor_product_vectorization.html" \l "equation-product) may be rewritten as**  **cij = ∑v=1p aiv bvj = ∑v=1p∑w=1n aiv δjw bvw = ∑v=1p∑w=1n (A⊗In)ij,vw bvwi=1,2,…,m; j=1,2,…,n.cij = ∑v=1p aiv bvj = ∑v=1p∑w=1n aiv δjw bvw = ∑v=1p∑w=1n (A⊗In)ij,vw bvwi=1,2,…,m; j=1,2,…,n.**  **which is equivalent to the matrix equation**  **▶Λmn(AB) = (A⊗In) ⋅ Λpn(B)▸Λmn(AB) = (A⊗In) ⋅ Λpn(B)**  **On the other hand, Eq. [(1)](http://visual.icse.us.edu.pl/LA/tensor_product_vectorization.html" \l "equation-product) yields also**  **cij = ∑w=1p bwj aiw = ∑v=1m∑w=1p δiv bTjw avw = ∑v=1m∑w=1p (Im⊗BT)ij,vw avwcij = ∑w=1p bwj aiw = ∑v=1m∑w=1p δiv bjwT avw = ∑v=1m∑w=1p (Im⊗BT)ij,vw avw**  **wherefrom we obtain an alternative relation for the matrix product  AB: AB:**  **▶Λmn(AB) = (Im⊗BT) ⋅ Λmp(A)** |